Name of Course	: **B.A. (Prog.) Mathematics**
Unique Paper Code	: **62354343**
Name of Paper	**: Analytical Geometry and Applied Algebra**
Semester	: **III**
Duration	: **3 hours**
Maximum Marks	: **75 Marks**

Attempt any four questions. All questions carry equal marks.

1. Describe, sketch and label the focus, vertex and directrix of the parabola

$$y^2 + 2y + 12x - 23 = 0.$$

Describe, sketch and label the centre, vertices, foci and asymptotes of the hyperbola

$$9x^2 - 4y^2 - 54x + 8y + 41 = 0.$$

2. Find the centre, vertices, foci and ends of minor axis of the ellipse

$$4x^2 + 9y^2 - 16x - 54y + 61 = 0$$

and sketch it.

Find equation of the parabola that has vertex at (1, 1) and focus (−3,1). What is its directrix?

Find equation of hyperbola having vertices $(\pm 3,0)$ and foci $(\pm 5,0)$.

3. Find a vector oppositely directed to $3i - 4j$ and $\frac{1}{3}rd$ the length of it. Also express $\mathbf{v} = 2i - 4j$ as sum of vector parallel and orthogonal to $\boldsymbol{a} = i + j$.

Rotate the coordinate axes to remove the '*xy*' term and identify the curve

$$x^2 - x\,y + y^2 - 2 = 0 \ .$$

Sketch $z = y^2$ in 3- space.

4. Find the equation of the sphere that is inscribed in the cube that has sides of length 4 and is parallel to coordinate planes also the sphere is centered at the point (−1, 0, 2).

Find the direction cosines of $\mathbf{v} = 2i + j - k$ and use them to find the direction angles. Also find the vector component of **v** along to $\mathbf{b} = j - k$.

Find the distance between the point $P(-4,0,1)$ and the line through the points $A(0, 0, -1)$ and $B(-3, 2, -3)$.

5. Find the parametric equation of line L passing through the points $(2,\ 4, -1)$, and $(5,\ 0,\ 7)$. Where does the line intersect the xy-plane?

Show that the line L: $x = 3 + 8t,\ y = 4 + 5t,\ z = -3 + t$ and the plane

$x - 3y + 7z = 12$ are parallel.

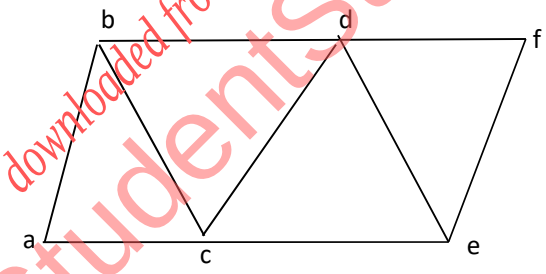
Show that the lines

$$L_1: x = 1 + 7t, \quad y = 3 + t, \quad z = 5 - 3t$$
$$L_2: x = 4 - t, \quad y = 6, \quad z = 7 + 2t$$

are skew. Also find the distance between them.

6. Define a Latin square. Give an example of a Latin square of order 6.

Find a minimal edge cover for the following graph. Give a detailed logical analysis.



Three pitchers of sizes 10 litres, 4 litres and 7 litres are given. If initially 10 litres pitcher is full and the other two empty, find a minimal sequence of pouring so as to have exactly 3 litres of water in two pitchers.